Title: Will Exams Be Canceled?

Brief Overview:

Students will develop a conceptual understanding of the characteristics of exponential functions using the graphing calculator. The graphing calculator will be used to generate data and their related graphs and perform regression analysis. The students will compare the graphs of polynomial functions and complete a model simulation.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will apply the process of mathematical modeling to discover growth and decay relationships as they exist in the real world. They will make and test conjectures based on data collected from a random number generator on the graphing calculator and draw conclusions based on comparisons of models. Students also will work cooperatively and discuss and compare their results. They will demonstrate real life situations and mathematical ideas, both orally and in writing. Last of all, students will represent data graphically, numerically and algebraically.

- **Patterns, Functions, and Algebra**
  Students will recognize exponential functions in graphic and symbolic form.

- **Data Analysis, Statistics, and Probability**
  Students will generate, organize and represent data in the context of exponential functions. They will interpret data and develop symbolic models.

Links to Virginia High School Mathematics Core Learning Units:

- **AII.8**
  The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

- **AII.15**
  The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.

- **AII.19**
  The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatter plots to determine the equation for a curve of best fit.

- **AII.8/T.8**
  The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

- **AII.15/T.15**
  The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.

- **AII.19/T.19**
  The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatter plots to determine the equation for a curve of best fit.

- **MA.1**
  The student will investigate and identify the characteristics of polynomial and rational functions and use them to sketch the graphs of functions. This will include determining zeroes, upper and lower bounds, y- intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

- **MA.8**
  The student will apply the techniques of translation and rotation of axes in the coordinate plane to graphing functions and conic sections. A graphing utility will be used to investigate and verify the graphs. Matrices will be used to represent transformations.

- **MA.9**
  The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and to solve equations and practical problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.

Links to National Science Education Standards:

- **Unifying Concepts and Processes**
  Students should develop an understanding of the concepts of evidence, models, and explanation.

- **Science as Inquiry**
  Students will participate in scientific investigations and formulate scientific explanations. They will reflect on the concepts that guide the inquiry. Students also will analyze evidence and data and communicate their explanations.

- **Science and Technology**
  Students will understand the relationship of data through the use of technology. They will be able to develop accurate concepts of the role and the relationship of science with technology interaction.

- **Science in Personal and Social Perspectives**
  Students will develop an understanding of population growth and the capacity of technology to improve environmental quality.

Links to Virginia High School Science Core Learning Units:

- **BIO.1**
  The student will plan and conduct investigations in which: graphing and arithmetic calculations are used in tools in data analysis; conclusions are formed based on recorded quantitative and qualitative data; alternative explanations and models are recognized and analyzed; and appropriate technology is used for gathering and analyzing data and communicating results.

- **BIO.9**
  The student will investigate and understand dynamic equilibria within populations, communities, and ecosystems. Key concepts include interactions within and among populations including carrying capacities, limiting factors, and growth curves.

Grade/Level:

Grades 9-12; Algebra II or higher; Biology or Chemistry

Duration/Length:

Four fifty-minute class sessions

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Characterization of linear, quadratic and higher polynomial functions
- Computation of regression models
- Use of a TI-83 graphing calculator

Objectives:

Students will be able to:

- graph exponential functions using real world phenomena.
- describe the characteristics of graphs that are polynomial functions.
- use a TI-83 graphing calculator to generate and analyze data simulated with the random binomial generator.

Materials/Resources/Printed Materials:

- TI-83 graphing calculator
- 100 dice or sugar cubes
- Container with lid
- Internet access (optional)
- Teacher notes
- Resource sheets 1–2
- Activity sheets 1–4
- Additional problems and quiz
- Teacher answer keys

Development/Procedures:

The students will need to have had prior experience with graphing functions, including linear, quadratic, cubic and quartic functions. The teacher will demonstrate a concrete presentation of the model so the students can visualize the involved in performing repeated trials. The class discussion will then lead to a more efficient way to generate the data of the model while simultaneously maintaining randomness of the data. Students' calculators will require the **FLU** program to simulate the model. Students will use a "seed" on their calculators so that all the data will be consistent. Students will examine and formulate conclusions about exponential functions through class discussion, completion of the simulation and other graphing activities, and varied assessment tools.

Assessment:

The teacher should conduct on-going assessment of student progress by circulating among cooperative student groups during class activities, and by evaluating student contributions to class discussion and questions. The teacher may also grade the activity sheets, additional problems and a quiz at the end of the learning unit. The scoring rubric is selective to the instructor requirements.

Extension/Follow Up:

- Students create their own hypothetical situation to model exponential decay.
- Students examine a radioactive decay model for Chemistry or conduct a CBL experiment such as measuring the change in pH of lemon juice with increasing amounts of antacids.
- Students perform a formal analysis of residuals from regression models in AP Statistics or AP Biology.
- Students can research the following web sites:

  1. http://score.kings.k12.ca.us/lessons/growth.html
  2. http://www.mathscribe.com
  3. http://www.earth.nwu.edu/people/seth/B02/decay.html
  4. http://www.exchangegame.com/ (note: a nominal fee of $.10 per participant is charged at the time of writing, July 1, 1999)

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Teacher's Notes

Before commencing the activities:
Review briefly types of functions that the class has studied previously (e.g., linear, quadratic and higher polynomial functions). Recall that regression models can be fit to both linear and nonlinear data using the statistics functions on the TI-83 graphing calculator.

Demonstration of the exponential decay model:
The teacher may perform this demonstration for the class prior to the calculator activities. Alternatively, time and materials permitting, students groups may perform the task.

Explain that a model will be demonstrated to show the spread of a disease in a student population of 100. Each die represents an individual student. A show of '6' on a die indicates that a student has been infected. It is not necessary to use or explain the terminology of exponential functions or decay models at this point.

Place 100 dice (or sugar cubes with one side marked with a colored dot). Shake the box well, then open the lid. Remove and count the number of dice that show a "6" (or the marked side of the sugar cube). These represent the number of infected students. Record the number of dice left in the box. These are the students who remain healthy. Record the data generated on the board or on an overhead in a table similar to the following:

Data Table

day	#healthy students	#infected students
0	100	17
1	83	14
2	69	7
…	…	…

Trials can be repeated until all dice have been removed from the box. Note the number of trials required.

Introduce the problem to be modeled:
Describe the situation the students will model in the epidemic activities. A flu epidemic is about to strike a school with 1000 students. The school term is approaching and exams are less than a month away. Today is Monday. The principal will call off exams if more than 50% of the student population is infected by the end of the week. The assumptions of the problem are that (1) there are 1000 healthy students at school prior to the epidemic; (2) the flu begins to infect students on Monday; (3) there is a 1/6 chance of infection each day; and (4) students are absent from school from the day after infection and they remain at home.

Ask the students how long they think it might take for everyone to get sick. Suggest that the class can simulate the spread of the disease using the TI-83 calculator with the **FLU** program. Time is saved if the students already have the **FLU** on their graphing calculator. It is also helpful if students clear the data in the lists on their calculator prior to using the **FLU** program.

Activity 1

Ask the class to form cooperative groups of 2-4 students. Ask student groups to work through **Activity 1**. Circulate around the classroom to listen to and interact with students in order to assess student progress. At the end of **Activity 1** ask students to complete all questions on the activity sheets to submit for grading or other form of evaluation. Orient the students to the next activity that follows, developing the mathematical ideas involved in the situation they have modeled in the simulation experiment.

Activity 2

Suggest to students to clear the **y =** editor. Use the calculator window setting as displayed on the activity sheet. To quickly check student answers, make a transparency and go over the answers on the overhead or, as an alternative, have all graphs on the teacher calculator and check using the TI viewscreen. Discuss with the students the differences in the graphs and reinforce the key terminology.

Activity 3

Students may work in the same cooperative groups as in **Activity 1** or they may form new groups. Ask student groups to work through **Activity 3**. In this activity student groups are permitted to select their own starting 'seed' value for the simulation. Groups who select either 'seed' = 2 or 9 will conclude that exams will take place. Those groups who select a 'seed' = 0, 1, 4-8 will conclude that exams will be called off. Remind students not to select a 'seed' = 3 because they already used that value in **Activity 1**. Circulate around the classroom to listen to and interact with students in order to assess student progress. At the end of **Activity 3** ask students to complete all questions on the activity sheets to submit for grading or other form of evaluation. Orient the students to the next activity that follows on extending the mathematical study of exponential functions.

Activity 4

The same basic instructions are used as in **Activity 2**. The objective of this activity is to have students understand transformations of exponential functions and their relation to changes in the elements of the general equation of the function. This concept can be related to composition of functions.

Extensions

Several possible extensions are described in the learning unit. Following is a description of the further mathematical and statistical analysis that can be done using the data generated in either **Activity 1** or **3**.

Students can examine the residuals corresponding to the linear versus exponential regression models. Calculate the residuals by subtracting the values predicted by the regression model from the observed data obtained in the simulation. Enter these residuals in a data list. Graph a scatter plot of the residuals (y-variable) versus the days (x-variable = L1). Examine the distribution of the residuals. Are the residuals evenly distributed over the entire time of the epidemic? Are certain regions over or underestimated? This extension can be used to compare the fit of regression models with different parameter values. Through this exercise, students will come to appreciate the differences between models and have further evidence to support their inferences about spread of a disease.

Students may point out that some of the assumptions of the model may not reflect a real life model of spread of a disease like the flu. Students can explore the distribution of data and the regression models obtained by varying the complexity of the assumptions. For example, in the simulation one assumption is that once a student becomes infected, they are removed from the population for the duration of the epidemic. Students can model that infected students become ill, stay home sick for a period of, say, 5 days, and then return to school. Students can model a biologically realistic situation in which once a student recovers from the illness and returns to school, they no longer can be infected because they have developed immunity to the disease.

Will Exams Be Canceled?

Activity 1

Group Member Names:
1. _________________ 3. _________________
2. _________________ 4. _________________

The Problem:
In communities in your area there has been an alarming number of cases of a virulent strain of the flu. The disease is about to strike your school! Students, parents and teachers are concerned because the end of the school term is approaching and exams are less than a month away. Today is Monday. Your principal has declared that if by the end of the week more than 50% of the school's student population of 1000 are absent due to this outbreak, then exams will be canceled.

Assumptions:
There are 1000 healthy students at school prior to the epidemic.
The flu begins to infect students on Monday.
There is a 1/6 chance of infection for every student each day.
Students are absent from school from the day after infection and they remain at home.

Hypothesis:
Given the assumptions above, discuss the problem with your group members and complete the statement:

We think that exams __________ (will/will not) be canceled due to the flu outbreak because ____.

Procedure:
1. Use the **FLU** program to generate simulated population data. So everyone obtains the same data, press **3 STO→ MATH PRB 1:rand ENTER ENTER**. To run the program, press **PRGM EXEC FLU ENTER**. It may take several minutes for the program to run. When the program has completed, **Done** will appear on your calculator's viewing screen.

2. Verify that the epidemic data is in lists L1 and L2 and confirm that everyone in your group has obtained the same data. To view the data lists, press **STAT EDIT 1:EDIT ENTER**; scroll down to view 24 pairs of values.

3. Record the data found in L1 and L2 in the table below. Calculate the number of students infected each day and enter those values in the table.

Data Table: Simulated Flu Epidemic

day L1	#healthy students L2	#daily flu cases
0	1000	
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
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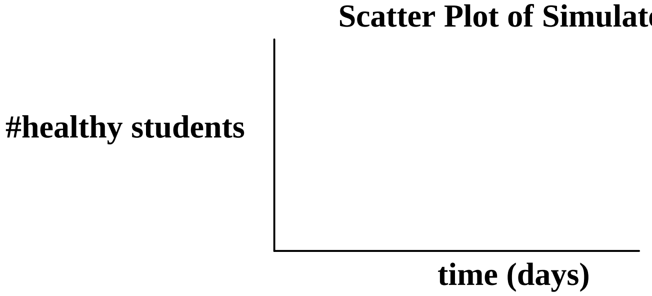
4. Graph a scatter plot of the flu data with the x-variable = L1 and the y-variable = L2. Refer to the **Resource 2** handout if you need help using the calculator.

   Sketch what you see on your calculator's viewing screen on the axes below.

Scatter Plot of Simulated Flu Epidemic



Describe the shape of the scatter plot. Is it linear or nonlinear? What is the behavior of the graph near the start of the epidemic? Near the end? Point out any outliers (data values that don't seem to fit the general pattern).

5. Compute a linear regression model for the simulated flu data.
   Refer to the **Resource 2** handout if needed.

   (a) Linear Regression Model
   From your calculator screen, record the model parameter values, a and b, rounding off to 3 decimal places.

   a = __________ b = __________

   (b) Evaluate the model. Press **ZOOM 9** to graph the simulated data in a scatter plot and the linear regression model simultaneously.

   Describe how well the linear regression model fits the flu epidemic data. Identify regions where the model works best. Where does the model underestimate the actual data values? Where does the model overestimate?

   (c) Examine the linear regression model parameters, a and b. How do you interpret a negative 'a' value? Explain what 'b' represents.

Analysis and Conclusions:
Discuss how you think the flu epidemic spreads. Include information you have gained by examination of the data, graphs and model you have obtained in this simulation.

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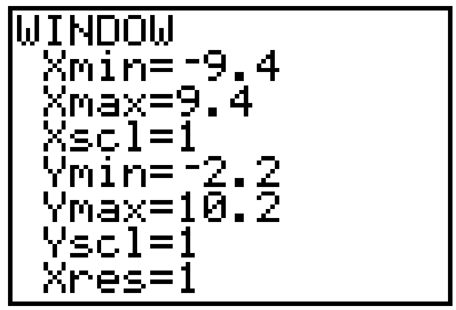
Activity 2 Name: ____________________

Objective: To determine the characteristics of basic exponential functions.

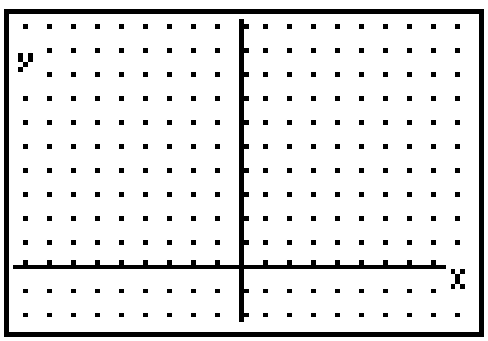
Definition: An exponential function is any function in the form $y = b^x$ where b is a constant and $b > 0, b \neq 1$. This is a basic exponential function.

Set your calculator window as follows:

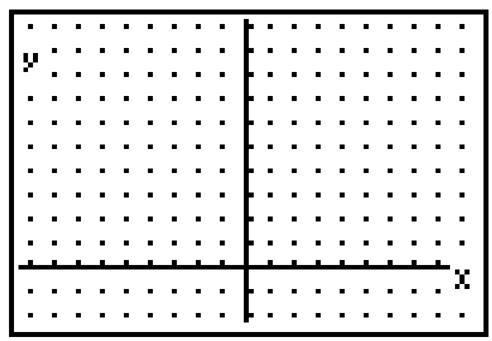


1. Graph each of the following on your calculator and on the axes below.

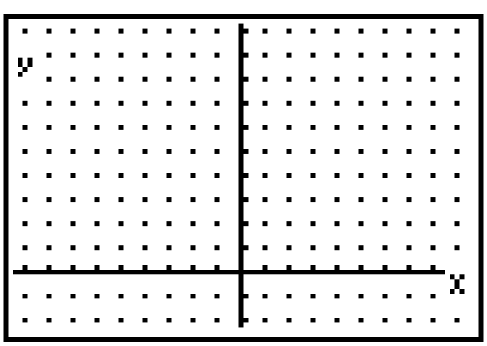
a. $y = 2^x$



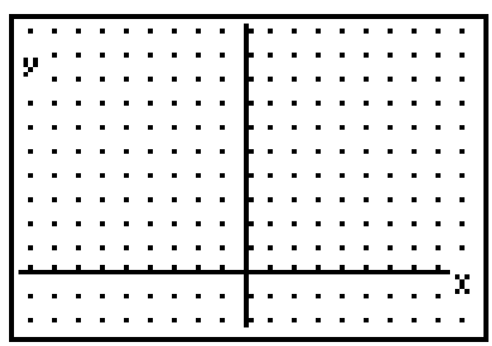
b. $y = e^x$



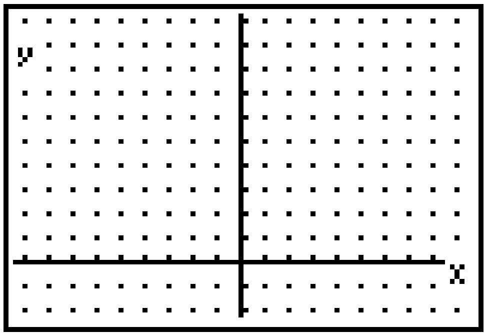
c. $y = 3^x$



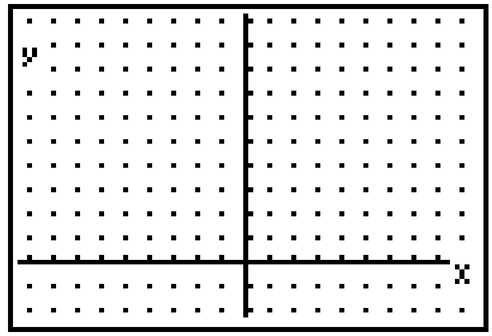
d. $y = 4^x$



e. $y = 5^x$



f. $y = 10^x$



2. Note any characteristics that are common to each of these graphs.

a. y-intercepts:

b. zeros:

c. symmetries:

d. apparent asymptotes:

e. increasing/decreasing (rising/falling):

f. concavity:

g. domain:

h. range:

3. Answer the following by looking at the graphs from Exercise 1.

a. What happens to the graph of $y = b^x$ as b gets larger?

b. Does $y = -2^x$ appear to be an exponential function? Why or why not?

c. Does $y = (-2)^x$ appear to be an exponential function? (You might want to change your window.) Why or why not?

d. According to the definition of an exponential function, b must be greater than 0. From your observations, why must this be so?

e. According to the definition of an exponential function, $b \neq 1$. Graph $y = 1^x$. From observing this graph, why does this restriction exist?

f. Graph $y = 2^{-x}$ and $y = \left(\frac{1}{2}\right)^x$. What do you see?

g. If I had the answer to a test question written on my answer key as $y = \left(\frac{2}{3}\right)^x$ and your answer was $y = \left(\frac{3}{2}\right)^{-x}$, would you receive full credit for your answer? Why or why not?

Will Exams Be Canceled?

Activity 3

Group Member Names:

1. ________________ 3. ________________
2. ________________ 4. ________________

The Problem Revised:

So far, you've examined the spread of the flu graphically and analyzed the data using linear regression models. You've found that the linear models do not seem to work very well. Is there a better model for the epidemic data?

The Assumptions:

There are 1000 healthy students at school prior to the epidemic.
The flu begins to infect students on Monday.
There is a 1/6 chance of infection for every student each day.
Students are absent from school from the day after infection and they remain at home.

Procedure:

1. Use the **FLU** program to generate simulated population data. So group members obtain the same data, decide on a starting 'seed' value. Select any whole number from 0-9 except for 3 that you used to obtain the data in **Activity 1**. Each group should use a different 'seed' number. Press the seed number key, followed by **STO→ MATH PRB 1:rand ENTER ENTER**. To run the program, press **PRGM EXEC FLU ENTER**. Again, it may take several minutes for the program to run. When the program has completed, **Done** will appear on your calculator's viewing screen.

2. Verify that the epidemic data is in lists L1 and L2 and confirm that everyone in your group has obtained the same data. To view the data lists, press **STAT EDIT 1:EDIT ENTER**; scroll down to view 24 pairs of values.

3. Record the data found in L1 and L2 in the table below.

Data Table: Simulated Flu Epidemic #2 Our group's starting 'seed' = ________

day L1	#healthy students L2
0	1000
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
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4. Graph a scatter plot of the flu data with the x-variable = L1 and the y-variable = L2. Refer to the **Resource 2** handout if you need help using the calculator.

   Sketch what you see on your calculator's viewing screen on the axes below.

Scatter Plot of Simulated Flu Epidemic #2

#healthy students

time (days)

Describe the ways in which the data and graph in this simulation compares with your results of the original simulation with ‘seed’ = 3.

5. This time, compute an exponential regression model for the simulated flu data. Refer to the **Resource 2** handout if needed.

(a) Exponential Regression Model
From your calculator screen, record the model parameter values, a and b, rounding off to 3 decimal places.

a = __________ b = __________

(b) Evaluate the model. Press **ZOOM 9** to graph the simulated data in a scatter plot and the exponential regression model simultaneously.

Describe the fit of the exponential model you obtained.

(c) Complete the Analysis and Conclusions questions and prepare to present your results and interpretations to the class.

Analysis and Conclusions:
Examine the exponential regression model parameters, a and b. Compare the parameter values to information given in the problem assumptions at the start of the activity. What value does ‘a’ seem to represent? How do you think the model value of ‘b’ relates to the problem assumptions?

Revise your thinking about how an infectious agent, such as the flu, spreads in a population. Include information you have gained by examination of the data, graphs and model you have obtained in the simulations.

For your group simulation, decide whether or not school exams will be canceled. Suggest a creative plan that your principal can implement to deal with the situation that would appeal to students, teachers and parents.

Will Exams Be Canceled?

Activity 4

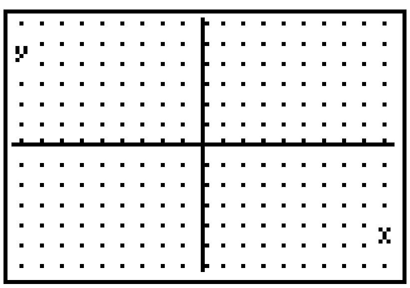
Objective: To determine translation patterns of exponential functions.

Set your calculator window as follows:

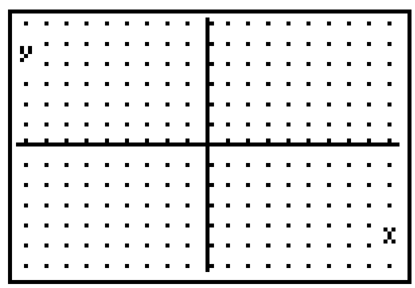
```
WINDOW
 Xmin=-9.4
 Xmax=9.4
 Xscl=1
 Ymin=-6.2
 Ymax=6.2
 Yscl=1
 Xres=1
```

1. Graph $Y_1 = 2^x$. Keeping that graph on the screen, graph a-f individually on the calculator and the axes below.

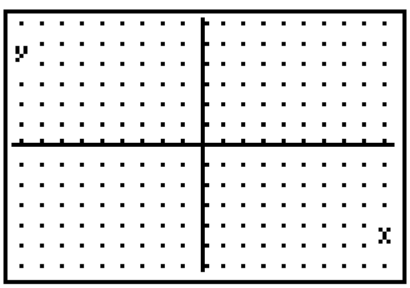
   a. $y = 2^{x-1}$



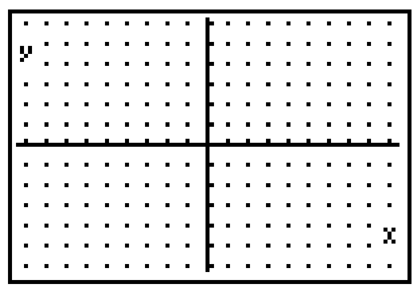
   b. $y = 2^{x+1}$



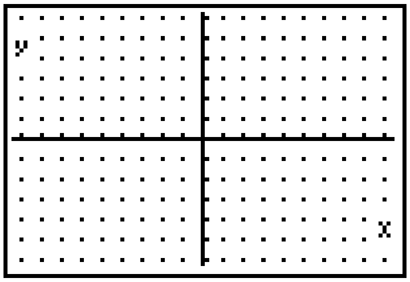
   c. $y = 2^x - 1$



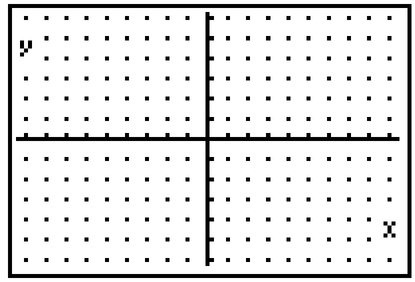
   d. $y = 2^x + 1$



e. $y = 2^{-x}$



f. $y = -2^{x}$

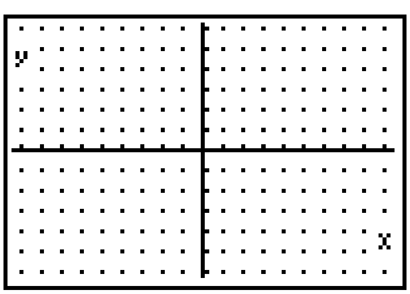


2. Compare the graphs in Exercise 1 to the graph of $y = 2^{x}$. Do any of the relationships you observe resemble the translation patterns of functions that we have already studied? If so, describe them in detail.

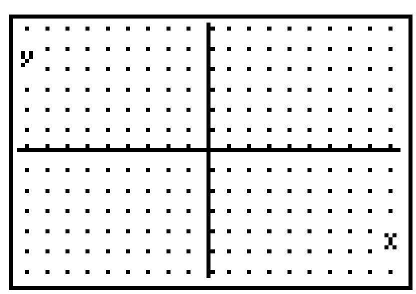
3. When you sketch a quick graph of an untranslated exponential function, what are two characteristics that anchor your graph?

4. Keeping the graph of $y = 2^{x}$ on your screen, graph each of the following on both your calculator and the axes below.

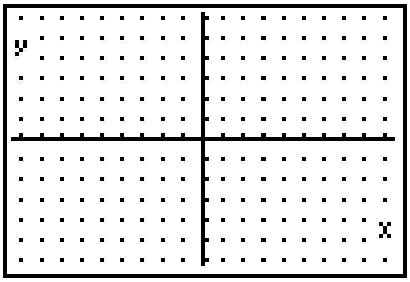
a. $y = 1.5 \cdot 2^{x}$



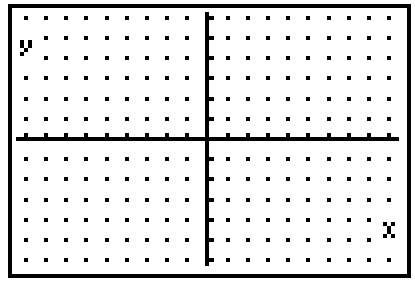
b. $y = 3 \cdot 2^{x}$



c. $y = 4 \cdot 2^x$



d. $y = \frac{1}{2} \cdot 2^x + 1$



5. Compare each of the graphs from Exercise 5 to the graph of $y = 2^x$.

   a. What major change do you observe?

   b. What about these equations causes this change?

   c. What did not change?

   d. Are these graphs translations of $y = 2^x$. Justify your answer.

e. What are the similarities between these equations and the regression equations we obtained from the flu problem?

f. How do these graphs differ from the graphs from the flu problem?

g. These equations are all in the form $y = a \cdot b^x$. What part(s) of these equations would you change to cause them to more closely resemble the flu graphs?

h. Exponential functions are frequently called “growth and decay” functions. Which type do you think the flu problem is? Why?

ANSWER KEY: ACTIVITY 1

Hypothesis:
Answers may vary, for example:
We think that exams __will_____ be canceled due to the flu outbreak because
the population reduces quickly since each day 1/6 of the remaining students get sick.

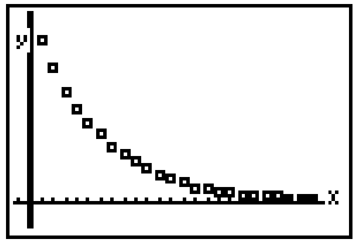
We think that exams __will not__ be canceled due to the flu outbreak because
1000 students is a large population, so it'd take a long time to reduce to 500.

Procedure:
3. Data Table: Simulated Flu Epidemic

day L1	#healthy students L2	#daily flu cases
0	1000	174
1	826	138
2	688	121
3	567	98
4	469	68
5	401	62
6	339	58
7	281	40
8	241	41
9	200	33
10	167	28
11	139	25
12	114	24
13	90	21
14	69	14
15	55	7
16	48	4
17	44	8
18	36	4
19	32	7
20	25	4
21	21	6
23	15	4
24	11	4
25	8	3

4.

Scatter Plot of Simulated Flu Epidemic



The shape of the scatter plot is nonlinear. The scatter plot drops down sharply at the beginning of the epidemic, curves in the middle, and flattens out toward the end. There do not appear to be any outliers.

5.
(a) Linear Regression Model

a = <u> - 29.170 </u> b = <u> 568.214 </u>

(b) The linear model does not adequately fit the data. Perhaps the linear model fits best in the middle of the epidemic but still it does not fit well. In this first simulation with a fixed starting 'seed,' the linear model underestimates at the beginning of the epidemic, overestimates somewhat in the middle, and underestimates at the end.
In fact, some students may notice that the linear model predicts negative values at the end of the graph that are inappropriate values (there can't be a negative number of students).

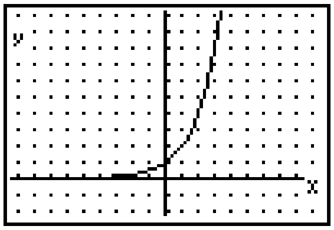
(c) In a linear model, 'a' represents the slope of the line. A negative value for 'a' is appropriate; the slope indicates that as time passes there are fewer healthy students remaining in school. In a linear model, 'b' represents the y-intercept of the line. The computed value of 568.214 is contrary to the starting assumptions of the epidemic model. The y-intercept should represent the number of students present at school prior to the epidemic, and should equal 1000.

Analysis and Conclusions:
Accept all reasonable and well-thought responses.

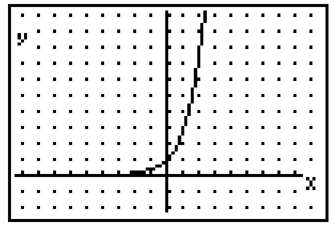
By the end of this activity, students should develop the understanding that in an epidemic there is a quick, dramatic effect on the population. Students should relate their conclusions to specific observations made on the data, graphs and the linear regression model.

ANSWER KEY: ACTIVITY 2

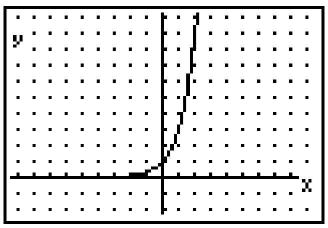
1. a.



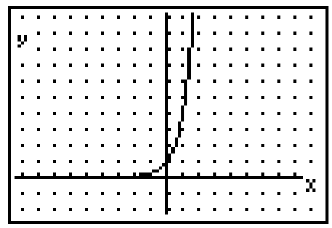
b.



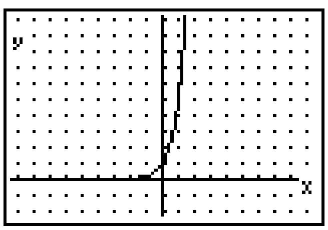
c.



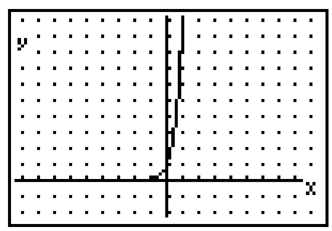
d.



e.



f.



2. a. (0, 1) for all
   b. none for all
   c. none
   d. horizontal: $y = 0$ for all
   e. all always increasing/rising
   f. all always concave up
   g. $\{x : x \in \Re\}$
   h. $\{y : y \geq 0\}$

3. a. The function rises more quickly or is steeper.
   b. Yes. It is a reflection of $y = 2^x$ across the *x*-axis.
   c. No. The definition of an exponential function restricts *b* to values greater than 0. (Also, all of the other examples are continuous functions. This one is not.)
   d. See above. Also, the points alternate between being above and below the *x*-axis.

(Although, you cannot tell this from the graph, points are missing for values of x such as 0.5. Try tracing the function.) This does not match the characteristics of exponential functions and will occur every time the base of the function is negative.

e. This function graphs as the horizontal line $y = 1$. The only characteristic that this function shares with the other exponential functions is its y-intercept, so it cannot be an exponential function.
f. Only one graph is visible. That means that the two functions graph the same and are equivalent.
g. Probably, because they are equivalent functions.

ANSWER KEY: ACTIVITY 3

3. Data Table: Simulated Flu Epidemic #2 Our group's 'seed' = varied

day L1	#healthy students L2
0	1000
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
23	
24	
25	

Student data tables should be complete. However, data will vary depending on starting 'seed,' but all members of a group should have the same

4. The shape should resemble an exponential function. Groups should observe that the data and graph generated with the second simulation exhibits differences that are expected to be slight. For example, one graph may decrease more sharply or flatten out more slowly.

5.
(a) Exponential Regression Model

a = 1074.207 b = 0.825

(b) The exponential regression model fits very well. The most obvious difference between the actual data and exponential regression model is the location of the y-intercepts.

Analysis and Conclusions:
In the exponential model, the parameter ‘a’ represents the number of students in the school prior to the start of the epidemic that equals 1000. In the exponential model, the parameter ‘b’ represents the chance of remaining healthy. In the problem assumptions, the chance of being infected equals 1/6, so the chance of remaining healthy equals 5/6 or 0.833.

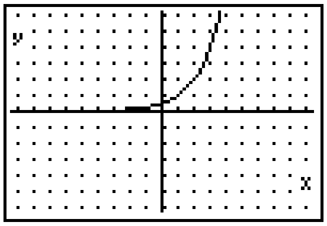
By the end of this activity, students should develop the understanding that an epidemic causes an exponential decrease in population size. Students should relate their conclusions to specific observations made on the data, graphs and the linear regression model.

Depending on the group data, exams may or may not be canceled. Check the student’s data table on day 4 (Friday, the end of the school week). If the number of healthy remaining students is less than 500, then exams will be called off. Groups who select either ‘seed’ = 2 or 9 will conclude that exams will take place. Those groups who select a ‘seed’ = 0, 1, 4-8 will conclude that exams will be called off. Students should not have used a ‘seed’ = 3 because they already used that value in **Activity 1**.

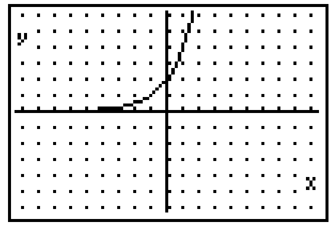
The suggested plan for the principal to implement should be reasonable and, hopefully, creative.

ANSWER KEY: ACTIVITY 4

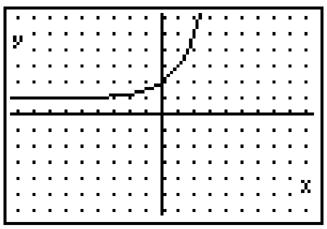
1. a.



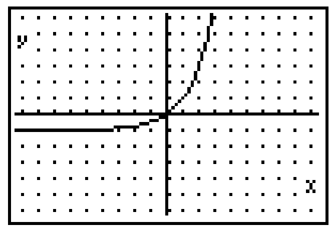
b.



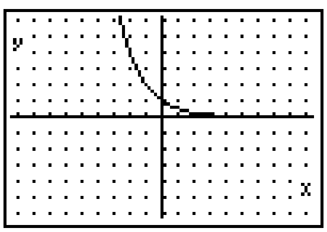
c.



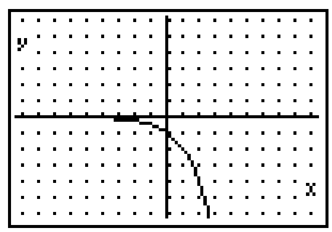
d.



e.



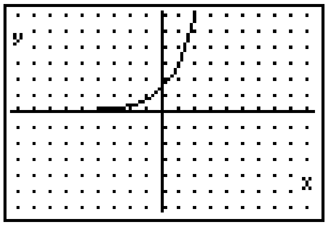
f.



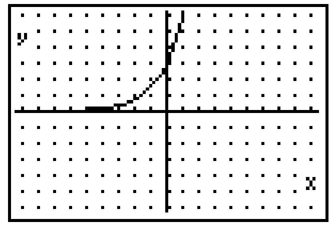
2. Yes, they all do. $y = 2^{x-1}$ and $y = 2^{x+1}$ are horizontal translations of $y = 2^x$. $y = 2^{x-1}$ moved right one unit and $y = 2^{x+1}$ moved left one unit. $y = 2^x - 1$ and $y = 2^x + 1$ are vertical translations of $y = 2^x$. $y = 2^x - 1$ moves down one unit and $y = 2^x + 1$ moves up one unit.
   $y = 2^{-x}$ is a reflection of $y = 2^x$ across the *y*-axis. $y = -2^x$ is the reflection across the *x*-axis.

3. The *y*-intercept and the horizontal asymptote $y = 0$.

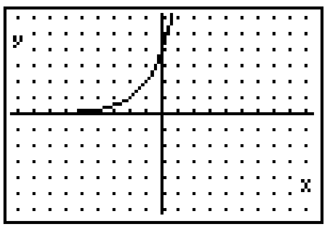
4. a.



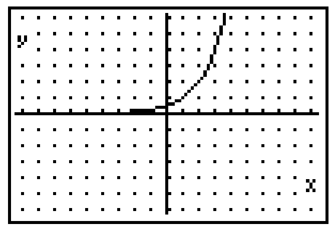
b.



c.



d.



5. a. The *y*-intercept is no longer (0, 1) (or (0, -1) for certain reflections.
   b. The coefficient of the basic function causes the change in the *y*-intercept. For instance, If $y = 3 \cdot 2^x$, at $x = 0$, $y = 3(1) = 3$. Therefore, the *y*-intercept is (0, 3) instead of (0, 1).
   c. The horizontal asymptote $y = 0$ remains the same.
   d. These are translations (dilations or contractions). If $f(x) = y = 2^x$, these are all in the form $y = af(x)$ where *a* is a constant.
   e. They are the same basic shape, have the same horizontal asymptote, and they all have *y*-intercepts different from (0, 1) or (0, -1).
   f. These graphs are all increasing functions. The function in the flu problem is decreasing.
   g. There are two ways to do this: change *b* to its reciprocal or change x to –x.
   h. The flu problem is a decay problem because the function is decreasing (the graph is falling) since the number of healthy people is decreasing everyday.

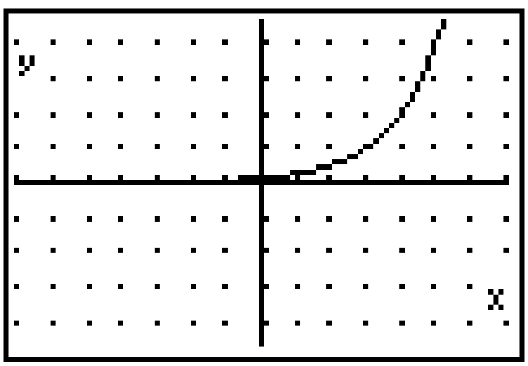
Will Exams Be Cancelled? Name:_________________________

Quiz

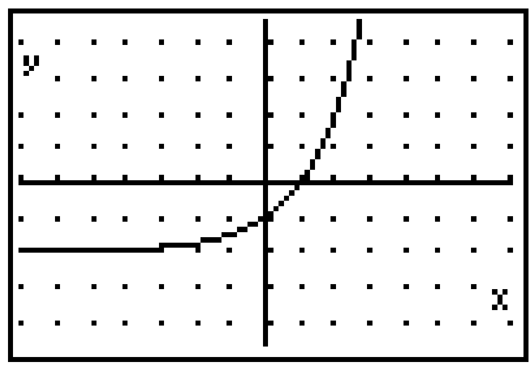
1. Consider the graph of the general exponential function $y = a^x$. Discuss its characteristics. In your discussion, include the terms domain, range, values of *a* for which the function is defined, asymptotes, symmetry, *x*-and *y*-intercepts, concavity, and increasing and/or decreasing.

QUESTIONS 2-5: The following graphs are translations of the graph of $y = 2^x$. In the blank provided, write the equation for each of the graphs below.

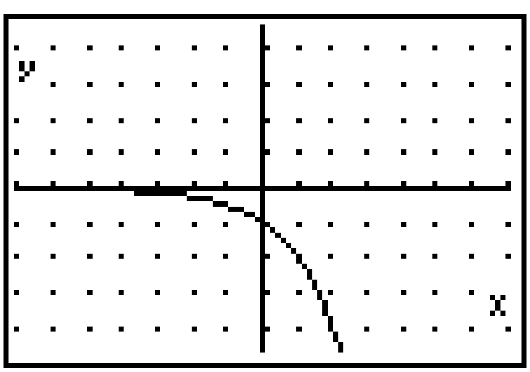
2. _____________________



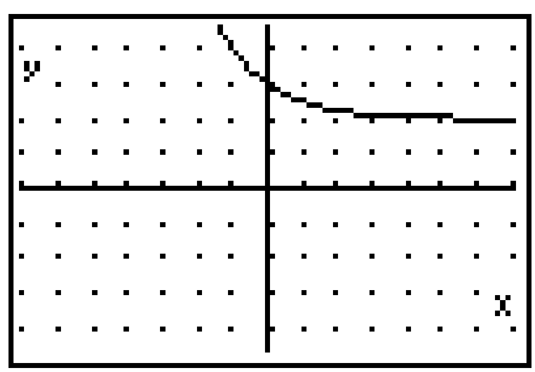
3. _____________________



4. _____________________



5. _____________________



QUESTIONS 6-7: Use the graphing calculator to answer the following.

6. Upon her birth Suzanne's grandparents opened a savings account for her by depositing $1000. The table below shows the yearly balance of the account from birth to age 11.

Yearly Balance

AGE	BALANCE
0	$1000.00
1	$1061.80
2	$1125.76
3	$1197.23
4	$1270.20
5	$1349.91
6	$1405.67
7	$1520.52
8	$1615.16
9	$1713.98
10	$1832.27
11	$1984.86

____________________a. Find an exponential regression equation to fit the data. Round to the nearest thousandth.

____________________b. Assuming that there are no additional deposits and no withdrawals, what will Suzanne's approximate balance be by the time she goes to college at age 18?

____________________c. According to the regression equation, what is the interest rate on the account?

____________________d. To the nearest year, how old will Suzanne be when the balance is $4000 (assuming no deposits or withdrawals)?

7. Mrs. Johnson attended a National Security Agency Summer Institute at Thomas Jefferson High School. She stayed at the home of a friend and drove to class everyday from Monday through Friday. In addition, on Sunday night, she drove to the school to make sure that she knew where it was located so that she would be on time for the start of class on Monday. Because her school system was to reimburse her for her mileage, she kept track of it on her car's odometer. Below is a table showing her daily mileage readings (rounded to the nearest mile). Sunday is day 0, Monday is day 1, etc.

Institute Mileage

DAY	MILEAGE
0	202
1	221
2	246
3	266
4	295
5	310

______________________a. Find an exponential regression equation to fit the data. Round to the nearest thousandth.

______________________b. If the class had been extended to two weeks, using your regression curve from Part a, predict Mrs. Johnson's mileage at the end of the tenth day.

______________________c. Find a linear regression equation to fit the data. Round to the nearest thousandth.

______________________d. If the class had been extended to two weeks, using your regression curve from Part c, predict Mrs. Johnson's mileage at the end of the tenth day.

______________________e. Which regression equation is the better model? Justify your answer in the space below.

8. A bacteria culture grows exponentially. The initial number of bacteria is 30. The table below indicates the amount of bacteria present at the beginning of each hour through the eleventh hour.

Bacteria Growth

HOUR	NUMBER
0	30
1	38
2	51
3	69
4	99
5	127
6	169
7	227
8	305
9	399
10	522
11	710

______________________a. Find an exponential regression equation to fit the data. Round to the nearest thousandth.

______________________b. About how many bacteria will be present at the beginning of the twenty-fourth hour?

______________________c. Abut when will there be 61,000 bacteria present?

Will Exams Be Cancelled?
Quiz Answer Key

1. The domain of the function is the set of real numbers. The range is $(0, +\infty)$. a must be greater than 0, but unequal to 1. There is a horizontal asymptote at $y = 0$. There are no obvious symmetries. The *y*-intercept is (0, 1); there is no *x*-intercept. The function is concave up over its entire domain. For *a* is between 0 and 1, the function is always decreasing; for *a* is greater than 1, the function is always increasing.

2. $y = 2^{x-3}$

3. $y = 2^{x} - 2$

4. $y = -2^{x}$

5. $y = 2^{-x} + 2$

6. a. $y = 994.609 \cdot 1.06^{x}$
   b. Using the trace command, the balance will be $2985.95. Using the rounded regression equation from Part a, the balance will be $2838.95.
   c. 6%
   d. Suzanne will be nearly 23 years old. (22.787311 with the regression equation in Y_1 and $Y_2 = 4000$ -- use the intersection command.)

7. a. $y = 205.813 \cdot 1.085^{x}$
   b. 465 miles (465.48893 miles using Trace)
   c. $y = 22.343x + 200.800$
   d. 424 miles (424,2381 miles using Trace)
   e. The linear regression is probably the better model because the mileage per day remained fairly constant. Any deviations from being constant could be due to difficulties in finding a parking space, taking a wrong turn, or running an errand on the way to or from class. (The first two things actually happened.)

8. a. $29.436 \cdot 1.336^{x}$
   b. About 31,022 (31021.824 using Trace)
   c. At about 26.3 hours (26.331591 by Intersect)

ADDITIONAL PROBLEMS

1. Suppose a colony of bacteria starts with 30 bacteria and the number increase at a particular rate per hour. Growth is recorded every hour for 9 hours and the data is as follows:

HOURS **(L1)**	# OF BACTERIA **(L2)**
0	30
1	42
2	53
3	72
4	91
5	126
6	168
7	224
8	299
9	400

   a. Use the **LIST** function on your calculator to graph the data in the table.
   b. Use a regression function to find a regression equation that describes this data.
   c. What kind of function is your regression equation?
   d. Using your regression equation, predict the number of bacteria present at the fifteenth hour.

2. Mrs. Johnson attended a summer math institute in Alexandria, Virginia. The mileage from home to her friend's house (where she stayed for the week) was 202.0 miles. When she arrived on Sunday, she drove to and from the conference location to make sure that she knew how to get there on time for the first class on Monday morning. Including Sunday's dry run, she made the trip six times. About the only mileage that she put on her car was from that daily trip. The table below contains her daily mileage data. (Sunday is day 0. Mileage is given to the nearest mile.)

DAY **(L1)**	MILEAGE **(L2)**
0	202
1	221
2	246
3	266
4	295
5	310

a. Use the **LIST** function on your calculator to graph the data in the table.
b. Use a regression function to find a regression equation that describes this data.
c. What kind of function is your regression equation?
e. If the class had lasted another week, approximate her mileage on day 10.

3. Jon put $1000 in a saving account that accrued interest continuously. The data table below shows how much money he had in the bank at the beginning of each year through the beginning of the tenth year. (The amounts in his account are given to the nearest ten cents.)

YEAR **(L1)**	AMOUNT IN ACCOUNT **(L2)**
0	$1000.00
1	$1061.80
2	$1127.50
3	$1197.20
4	$1271.20
5	$1349.90
6	$1433.30
7	$1522.00
8	$1616.10
9	$1716.00
10	$1822.10

a. Use the **LIST** function on your calculator to graph the data in the table.
b. Use a regression function to find a regression equation that describes this data.
c. What kind of function is your regression equation?
d. Approximate the amount in Jon's account at the beginning of the twentieth year.

.

ADDITIONAL PROBLEMS: ANSWERS

1. a. check on calculator
   b. $y = 30.24144423 * 1.331277844^x$
   c. exponential
   d. x = 15

2. a. check on calculator
   b. $y = 21.78571429x + 199.9571429$
   c. linear
   d. x = 10

3. a. check on calculator
   b. $y = 999.9895084 * 1.06187583^x$
   c. exponential
   d. x = 20 years

Will Exams Be Canceled?

Resource 1: Epidemic Simulation Program for the TI-83 Calculator

Note: The teacher should enter the program into his or her calculator as follows:

Press **PRGM**
NEW
1: Create New
Name = FLU

See note below before beginning to enter the program in the calculator.

Enter the program steps:

```
:1000→C
:seq(x, x, 1, 24)→L1
:For(A, 1, 24, 1)
:C-randBin(C, (1/6))→L2(A)
:L2(A)→C
:End
```

Press **2nd QUIT** to exit the program editor.

Note: **seq** is found under **2nd LIST OPS 5:seq(** ; **For** and **End** can be found under **PRGM CTL**; **randBin** can be found under **MATH PRB 7:**). In line 4 of the program, the '-' represents the subtraction sign.

Share the program **FLU** with each student by using a graphing calculator cable link.
Press **2nd LINK**
Teacher: Press **SEND 3:Prgm FLU ENTER**
Student: Press **RECEIVE ENTER**

The program has been successfully transferred when **Done** appears on the student's viewing screen.

To run **FLU** press **PRGM EXEC FLU ENTER**. Running time for the program **FLU** is expected to be several minutes. When the program is completed, Done will appear on the graphing calculator's viewing screen. After running the program, verify that the epidemic data have been stored in L1 (time in days) and L2 (number of students remaining healthy). To view the data lists, press **STAT EDIT 1:EDIT**. Scroll down to view all 24 pairs of values.

Will Exams Be Canceled?

Resource 2: Plotting Statistical Graphs and Computing Regression Models with the TI-83 Calculator

Example: To obtain a scatter plot of data

Be sure that calculator is **ON**.

Press **2nd STAT PLOT 1:Plot1 ON**
Type: first choice is scatter plot
Xlist: L1
Ylist: L2
Mark: choose the square
ZOOM 9

The scatter plot should appear on the viewing screen.

Example: To obtain a linear regression model

Be sure that calculator is **ON**.

Press **STAT CALC 4:LinReg(ax + b)**

LinReg(ax + b) should appear on the viewing screen. Various other regression models can be computed, including a quadratic model (5:QuadReg) and an exponential model (0:ExpReg).

Press **2nd L1 , L2 , VARS Y-VARS 1:Function ENTER 1:Y1 ENTER ENTER**

A linear regression model should appear on the calculator's viewing screen.
Record the model and verify that the model is stored in **Y= Y1=**

To graph the scatter plot of the data obtained in the epidemic simulation with the program **FLU** along with the regression model computed:

Press **2nd STAT PLOT 1:Plot1 ON ZOOM 9**

Examine visually and describe the fit of the regression model to the data.